5.4 Polynomial and Rational Inequalities

Learning Objectives

1. Solve Polynomial Inequalities
2. Solve Rational Inequalities
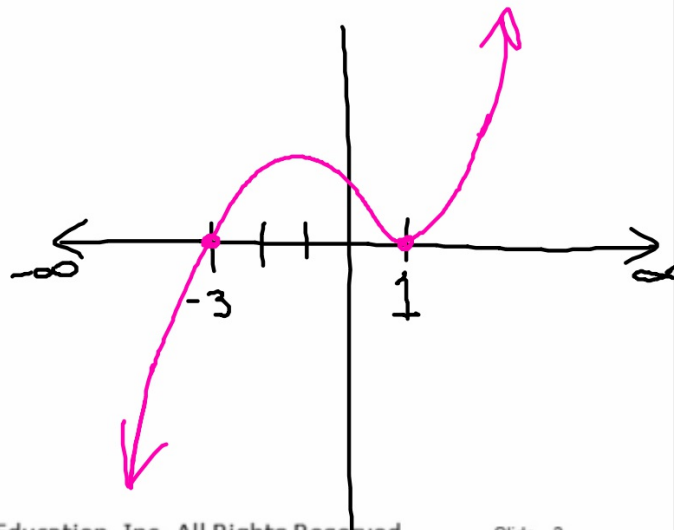
(graphically and
algebraically)

Example 1

Solving a Polynomial Inequality Using Its Graph

Solve $(x+3)(x-1)^2 > 0$ by graphing $f(x) = (x+3)(x-1)^2$.

Solution set:

$$\left(-3, 1\right) \cup \left(1, \infty\right)$$

Example 2

How to Solve a Polynomial Inequality Algebraically

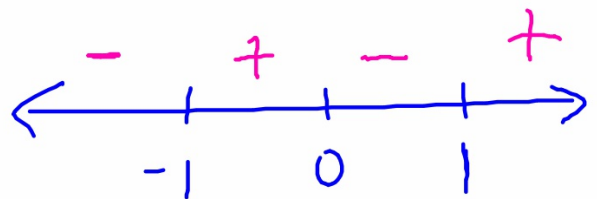Solve the inequality $x^3 > x$ algebraically. Answer using interval notation.

Draw a line and label the x-ints.

$$x^3 > x$$

$$x^3 - x > 0$$
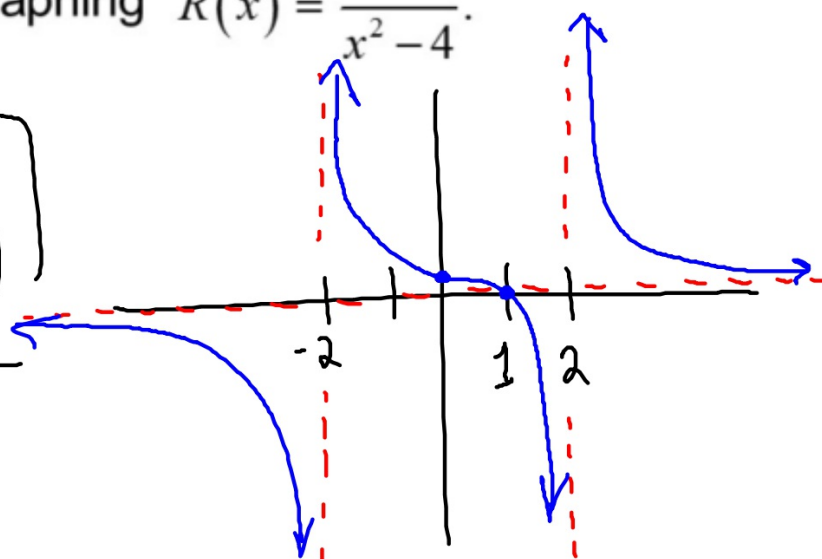
$$x(x^2 - 1) > 0$$

$$x(x+1)(x-1) > 0$$

Solution: $(-1, 0) \cup (1, \infty)$

Ⓟ Pearson

Example 3

Solving a Rational Inequality Using Its Graph

Solve $\dfrac{x-1}{x^2-4} \geq 0$ by graphing $R(x) = \dfrac{x-1}{x^2-4}$.

Solution

$(-2, 1] \cup (2, \infty)$

Example 4

How to Solve a Rational Inequality Algebraically

Solve the inequality $\dfrac{x^2+5x-2}{x-5} \leq 1$ algebraically.

Answer using interval notation.

$$\frac{x^2+5x-2}{x-5} \leq 1$$
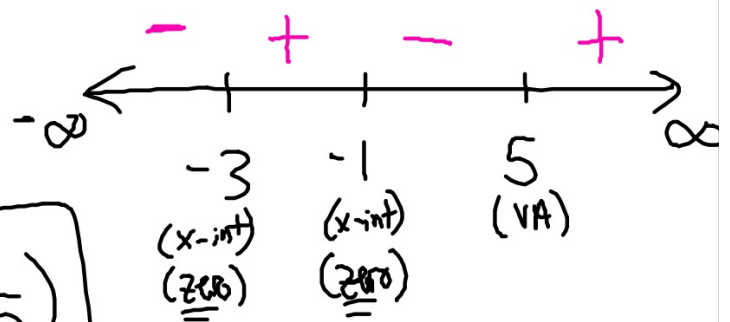
$$\frac{x^2+5x-2}{x-5} - 1 \leq 0$$

$$\frac{x^2+5x-2}{x-5} - \frac{x-5}{x-5} \leq 0$$

$$\frac{x^2+4x+3}{x-5} \leq 0$$

$$\frac{x^2+4x+3}{x-5} \leq 0$$

Draw a number line
and label x-ints.
and V.A.

$$\frac{(x+1)(x+3)}{x-5} \leq 0$$

Solution set

$$\boxed{(-\infty,-3] \cup [-1,5)}$$

$$-\infty \overset{\textcolor{magenta}{-}\quad\textcolor{magenta}{+}\quad\textcolor{magenta}{-}\quad\textcolor{magenta}{+}}{\longleftarrow\!\!|\!\!\underset{-3}{}\!\!|\!\!\underset{-1}{}\!\!|\!\!\underset{5}{}\!\!\longrightarrow} \infty$$

-3
(x-int)
(zero)

-1
(x-int)
(zero)

5
(VA)